Name and Surname : SOLUTIONS

Grade/Class : 11/..... **Mathematics Teacher** : .....

June Examination

ANSWER BOOKLET

PHASE 1	/ 75

QUESTION 1

1.1.1	$x(6-x)=0$	
	$x=0$ ✓ or $x=6$ ✓	2
1.1.2	$3x^2-2x-6=0$	
	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	
	$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(6)}}{2(3)}$ ✓	
	$x=-1,12$ ✓ or $x=1,79$ ✓	3
1.1.3	$(3-x)(7+x)<0$ ✓ C.V	
	C.V $x=3$ or $x=-7$	
	$\ominus \quad 0 \quad + \quad 0 \quad \ominus$ $-7 \qquad 3$	
	$\therefore\ x<-7$ or $3<x$ ✓✓ or 0	3

1.1.4 $\sqrt[3]{32} = 8^{3x} \cdot 2^{6x}$

$2^{\frac{5}{3}} = (2^3)^{3x} \cdot 2^{6x}$ ✓ p.b

$2^{5/3} = 2^{15x}$ ✓

$\frac{5}{3} = 15x$

$x = \frac{1}{9}$ ✓ 3

1.1.5 $x - 4 - 2\sqrt{x-1} = 0$

$x - 4 = 2\sqrt{x-1}$ ✓ isolate

$(x-4)^2 = (2\sqrt{x-1})^2$ ✓ sq

$x^2 - 8x + 16 = 4(x-1)$

$x^2 - 12x + 20 = 0$ ✓ std form

$(x-10)(x-2) = 0$ ✓ fact 5

$x = 10$ or $x = 2$

Check Soln

$x \neq 2$ ∴ $x = 10$ ✓ ans selection

reject

1.1.6 $3x^{\frac{2}{5}} - 8 = 0$

$x^{2/5} = \frac{8}{3}$ ✓ isolate

$(x^{2/5})^{\frac{5}{2}} = \pm(\frac{8}{3})^{\frac{5}{2}}$ ✓ raise both sides rec

$x = \pm 11,61$ ✓ ans ± 3

1.2.1 $= 4.3^{1-x} + 3^{2-x}$

$= 4.3^{1}.3^{-x} + 3^{2}.3^{-x}$ ✓ split

$= 3^{-x}(4.3 + 3^{2})$ ✓ c.f

$= 3^{-x}(12 + 9)$

$= 3^{-x}.21$ ✓

$= \frac{21}{3^{x}}$ Shown. 3

1.2.2 $4.3^{1-x} + 3^{2-x} = 63$

$\frac{21}{3^{x}} = 63$ ✓ equate

$21 = 63.3^{x}$

$\frac{21}{63} = 3^{x}$ ✓ Isolate

$\frac{1}{3} = 3^{x}$

$3^{-1} = 3^{x}$

$-1 = x$

$\therefore x = -1$ ✓ ans 3

1.3 $x^2 + 2yx = 3y^2$ and $2y - x = 6$

$x = 2y - 6$ ✓

$(2y-6)^2 + 2y(2y-6) = 3y^2$ ✓

$(2y-6)(2y-6) + 2y(2y-6) = 3y^2$

$4y^2 - 24y + 36 + 4y^2 - 12y - 3y^2 = 0$

$5y^2 - 36y + 36 = 0$ ✓

$(5y-6)(y-6) = 0$ ✓

$y = \frac{6}{5}$ or $y = 6$ ✓ both

$x = 2y - 6$ or $x = 2y - 6$

$x = 2(\frac{6}{5}) - 6$ $\quad x = 2(6) - 6$

$x = -\frac{18}{5}$ $\quad x = 6$ ✓ both

b

1.4 $\sqrt[p]{\frac{10^p + 2^{p+2}}{5^{2p} + 4.5^p}}$

$= \left[\frac{2^p \cdot 5^p + 2^p \cdot 2^2}{5^p \cdot 5^p + 4.5^p}\right]^{\frac{1}{p}}$ ✓ rational exp. ✓ split

$= \left[\frac{2^p(5^p + 4)}{5^p(5^p + 4)}\right]^{\frac{1}{p}}$ ✓ c.f.

$= \left(\frac{2}{5}\right)^{p(\frac{1}{p})}$

$= \frac{2}{5}$ ✓ ans

4.

QUESTION 2

2.1

$$\sqrt{3}\left(\sqrt{12}-\sqrt{1\tfrac{1}{3}}\right)$$

$\sqrt{4.3} = 2\sqrt{3}$ ✓

$\sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$ ✓

$$= \sqrt{3}\left(\sqrt{4.3}-\sqrt{\tfrac{4}{3}}\right)$$

$$= \sqrt{3}\left(2\sqrt{3}-\frac{2}{\sqrt{3}}\right)$$

$$= 2.3 - 2$$

$$= 4 \quad \checkmark$$

3

2.2

$$\frac{6}{\sqrt{3}+3} = a + b\sqrt{3}$$

$$\frac{6}{\sqrt{3}+3} \times \frac{\sqrt{3}-3}{\sqrt{3}-3} \quad \checkmark \text{ rat}$$

$$= \frac{6(\sqrt{3}-3)}{3-9}$$

$$= \frac{6\sqrt{3}-18}{-6} \quad \checkmark \text{ simplify}$$

$$= -\sqrt{3} + 3$$

$$= 3 - \sqrt{3}$$

4

$\therefore$ $a = 3$ ✓ and $b = -1$ ✓

2.3	$3^x = 5$	
	$(3^x)^y = 5^y$	
	$3^{xy} = 7$ ✓	
	$(3^{xy})^z = (7)^z$	
	$3^{xyz} = 9$ ✓	
	$3^{xyz} = 3^2$	
	$\therefore xyz = 2$ ✓	3
	NB WOC!!	

QUESTION 3

3.1	$x = -3$ or $x = \frac{5}{4}$	
	$(x+3)(4x-5) = 0$ ✓ Sub in	
	$4x^2 + 7x - 15 = 0$ ✓	2
3.2	$x^2 - 5x + c = 0$	
	roots $x = \frac{5 \pm \sqrt{41}}{2}$	
	$\Delta = b^2 - 4ac$	
	$\Delta = (-5)^2 - 4(1)(c)$ ✓ Sub in	
	$= 25 - 4c$ ✓ Δ	
	but $\Delta = 41$	
	$25 - 4c = 41$ ✓ equate	
	$-4c = 41 - 25$	
	$-4c = 16$	4
	$c = -4$ ✓ ans	

3.3 $x^2 + kx = k + 2x$

$x^2 + kx - 2x - k = 0$

$x^2 + (k-2)x - k = 0$

$a = 1 \quad b = k-2 \quad c = -k$

$\Delta = b^2 - 4ac$

$= (k-2)^2 - 4(1)(-k)$ ✓ Sub in

$= k^2 + 4$ ✓ Δ

∴ Roots are real and unequal ✓

for all $k \in \mathbb{R}$ $k^2 \geq 0$

$k^2 + 4 \geq 4$

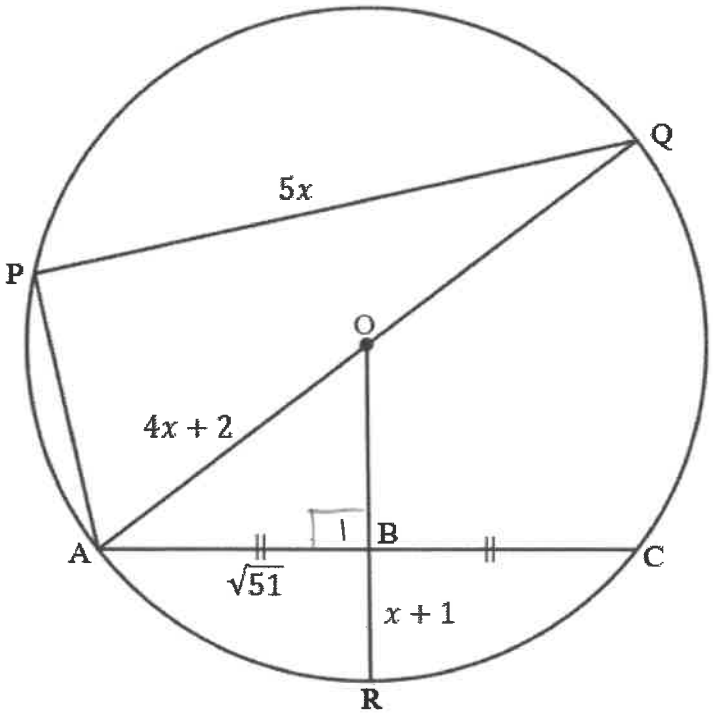
$k^2 + 4 > 0$

$\Delta > 0$ ✓ argument

4

QUESTION 4

4.



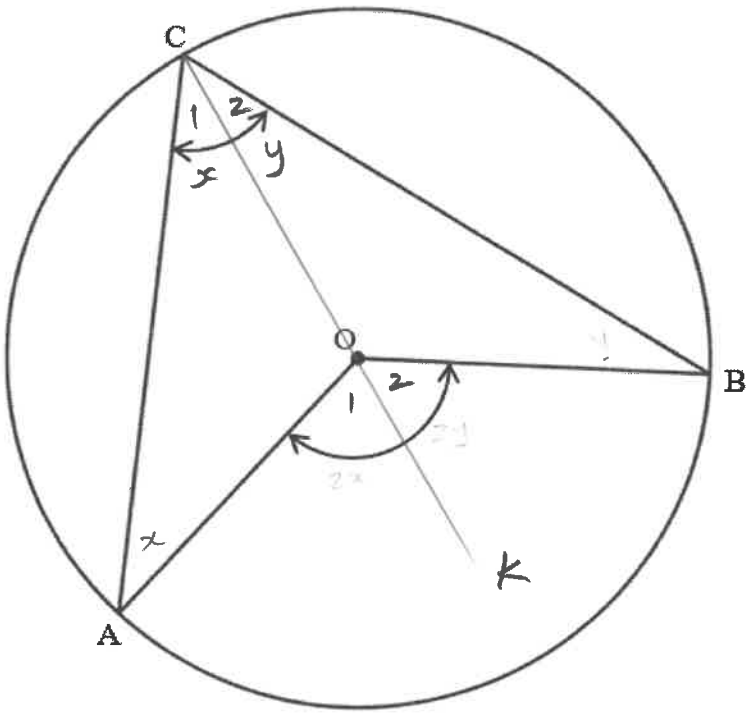
SR

4.1.	$\hat{B}_1 = 90°$ ✓ line from centre O to midpt chord	
	$OR = 4x + 2$ radii	
	$\therefore OB = 4x + 2 - (x + 1)$	
	$= 4x + 2 - x - 1$	
	$= 3x + 1$ ✓S	
	$(\sqrt{51})^2 + (3x+1)^2 = (4x+2)^2$ Pythag ✓SR	
	$51 + 9x^2 + 6x + 1 = 16x^2 + 16x + 4$	
	$0 = 7x^2 + 10x - 48$ ✓	
	$= (x - 2)(7x + 24)$ ✓	
	$\therefore x = 2$ or $-\frac{24}{7}$ reject	5

4 2.	$OA = 2(4x+2)$ radii	
	$= 8x + 4$	
	$= 8(2) + 4$	
	$= 20$	
	$PQ = 5x$ given	
	$= 5(2)$	
	$= 10$	
	$\hat{P} = 90°$ ✓SR. ^ in semi ⊙ $= 90°$	
	$PA^2 + 10^2 = 20^2$ ✓S Pythag	
	$PA^2 = 300$	
	$PA = \pm\sqrt{300}$ reject −	
	$= 17,32$ ✓ans 2 dp	3

QUESTION 5

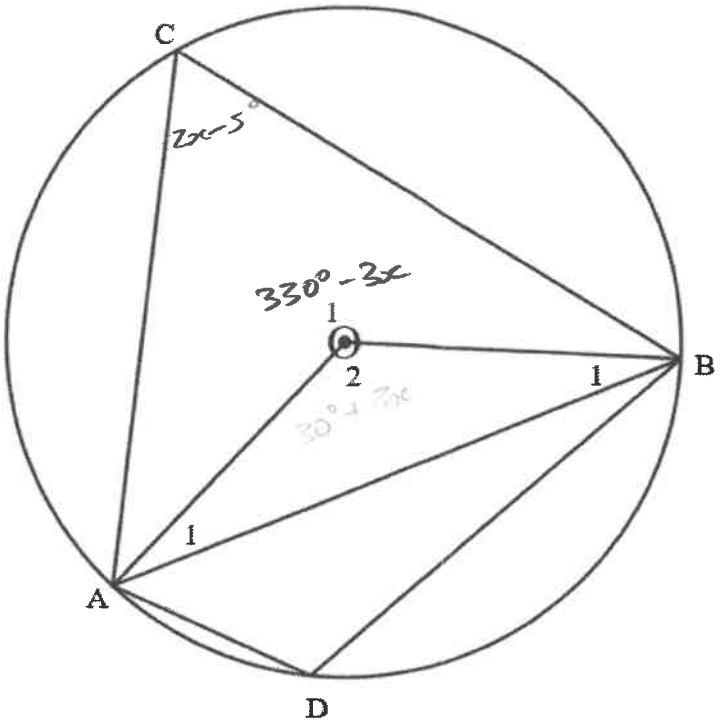
5.1.



✓ constr

	Let $\hat{C}_1 = x$ and $C_2 = y$	
	$OC = OA$ ✓SR radii	
	$\hat{A} = x$ ^'s opp = sides	
	$\therefore \hat{O}_1 = 2x$ ✓SR ext ^ Δ	
	Similarly, $\hat{O}_2 = 2y$ ✓S	
	$\therefore AOB = \hat{O}_1 + \hat{O}_2$	
	$= 2x + 2y$ ✓ conclusion	
	$= 2(x+y)$	
	$= 2(\hat{C}_1 + \hat{C}_2)$	
	$= 2\ A\hat{C}B$	5

5.2.



5 2.1.	$\hat{O}_2 + 330° - 3x = 360°$ ∠'s in a rev = 360°	
	$\hat{O}_2 = 30° + 3x$ ✓SR	
	$30° + 3x = 2(2x - 5°)$ ✓S ✓R ∠@ centre = 2× ∠@ circum	
	$30° + 3x = 4x - 10°$	
	$40° = x$ ✓ans	4
5.2.2.	Let $\hat{B}_1 = y$	
	$\hat{A}_1 = \hat{B}_1 = y$ ✓SR radii, ∠'s opp = sides	
	$2y + 30° + 3(40°) = 180°$ ✓SR sum ∠'s in $\Delta = 180°$	
	$y = 15°$	
	$\therefore \hat{B}_1 = 15°$ ✓ans	3

5.2 OR

5.2.1. $\hat{D} + 2x - 5^\circ = 180^\circ$ opp $\angle$'s cyclic quad $= 180^\circ$

$\hat{D} = 185^\circ - 2x$

$330^\circ - 3x = 2(185^\circ - 2x)$ $\angle$ @ centre $= 2 \times \angle$ @ circum

$= 370^\circ - 4x$

$x = 40^\circ$

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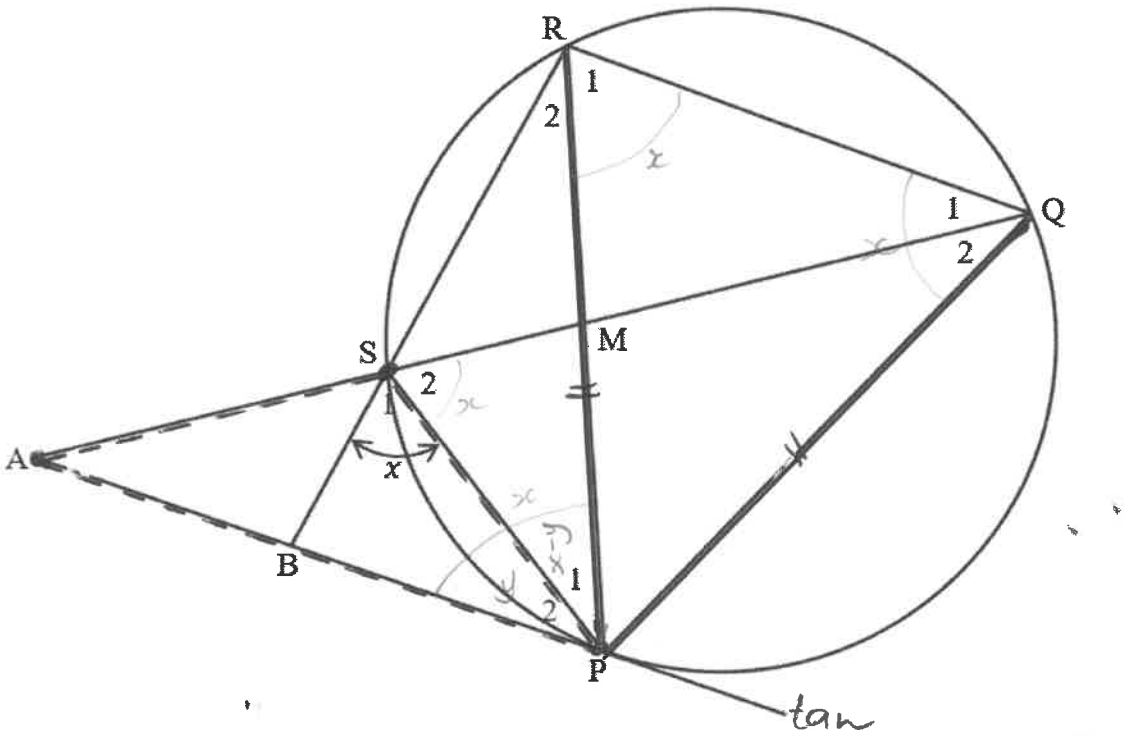
QUESTION 6

6.



6.1.	$\hat{Q}_1 + \hat{Q}_2 = x$ ✓S ✓R	ext ^ cyclic quad	
	$\hat{R}_1 = x$ ✓SR	^'s opp = sides	
	$\hat{S}_2 = x$ ✓S ✓R	^'s in same ⊙ segm =	
	$\therefore \hat{S}_1 = \hat{S}_2$	both = x	5
4 2.	let $\hat{P}_2 = y$		
	$\hat{P}_1 + \hat{P}_2 = x$ ✓S ✓R	tan chord thm	

$\therefore \hat{P}_1 = x - y$ ✓S

$\hat{S}_2 = \hat{A} + \hat{P}_2$ ext ∠ Δ

$x = A + y$

$x - y = \hat{A}$ ✓SR

$\therefore \hat{P}_1 = \hat{A}$ ✓S both = $x - y$

∴ PM is a tan ✓R → conv tan chord thm

to ⊙ PSA

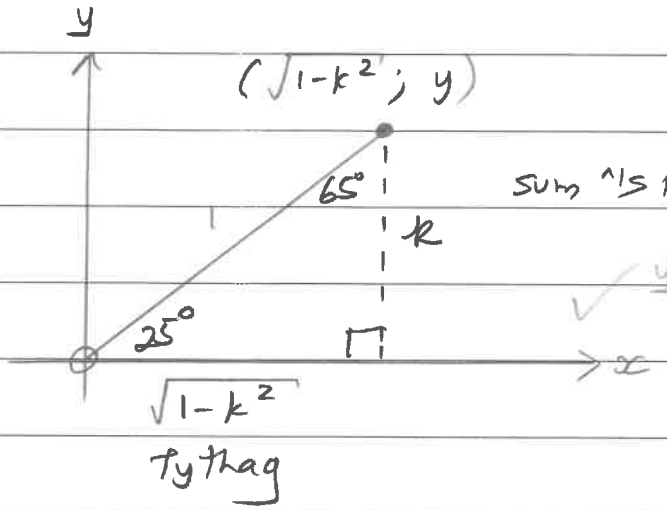
@ P

6

QUESTION 7

7.1.1. $\sin 25° = k$

$= \frac{k}{1} \quad \frac{y}{r}$



✓ y, r

$\tan 25° = \frac{k}{\sqrt{1-k^2}}$ ✓ $\frac{y}{x}$ 2

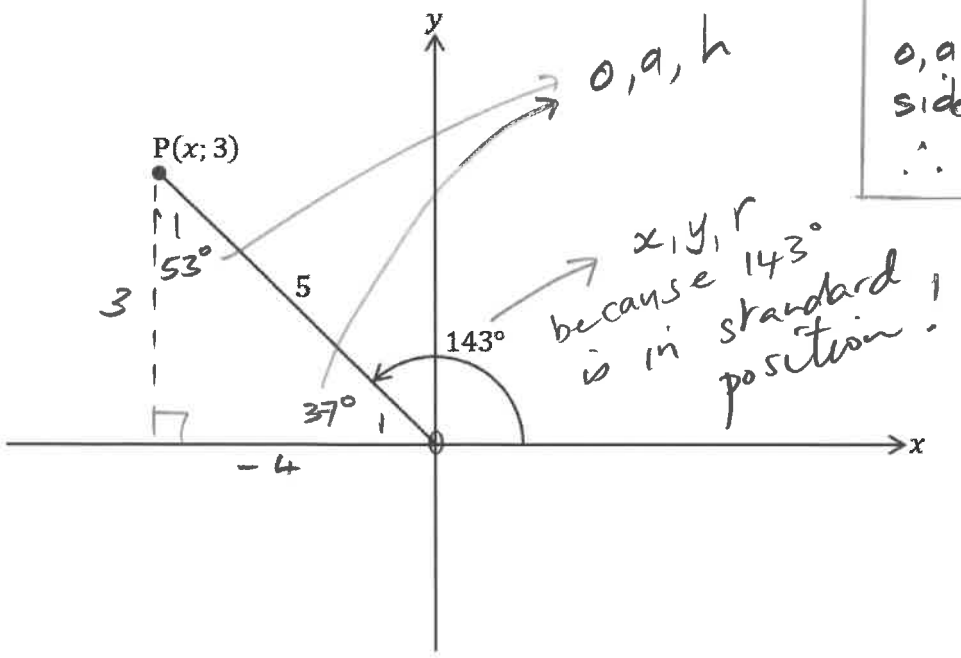
7.1.2. $\cos(-25°)$

$= \cos 25°$ ✓ red

$= \frac{\sqrt{1-k^2}}{1}$ $\frac{x}{r}$

$= \sqrt{1-k^2}$ ✓ ans 2

7.2.



o, a, h

x, y, r because 143° is in standard position!

NB

o, a, h are the sides of a Δ

∴ +

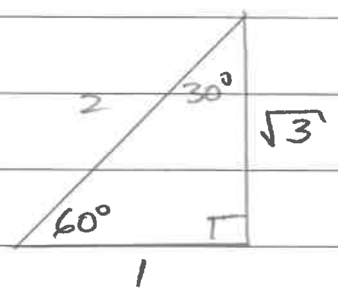
7.2.1.	$x^2 + 3^2 = 5^2$ Pythag	
	$x^2 = 16$	
	$x = \pm\sqrt{16}$ reject +	
	$= -4$ ✓S	1
7.2.2.	$\sin 143°$	
	$= \frac{3}{5}$ ✓ $\frac{y}{r}$	1
7.2.3.	$\hat{O}_1 = 37°$ ∠'s on str line $= 180°$	
	$\cos 37°$	
	$= \frac{4}{5}$ ✓ ✓ $\frac{a}{h}$	2

7.2.4. $\hat{P}_1 = 53^\circ$ sum ∠s in Δ = 180°

$\tan 53^\circ$

$= \frac{4}{3}$ ✓✓ $\frac{o}{a}$ 2

7.3.1.



✓   1

7.3.2. $\tan 150^\circ = \tan(180^\circ - 30^\circ)$

$= -\tan 30^\circ$ ✓ red

$= -\frac{1}{\sqrt{3}}$ ✓ $\frac{o}{a}$ 2

7.4 $\frac{\sin^2(x-180^\circ) + \frac{\tan 197^\circ}{\tan 343^\circ}}{1 + \cos(1530^\circ + x)}$

• $\sin^2(x-180^\circ) = [\sin(x-180^\circ)]^2$

$= [\sin(x+180^\circ)]^2$ $+360^\circ$

$= [\sin(180^\circ + x)]^2$

$= [-\sin x]^2$

$= \sin^2 x$

	• $\tan 197^\circ = \tan(180^\circ + 19^\circ)$	
	$= \tan 17^\circ$	
	• $\tan 343^\circ = \tan(360^\circ - 17^\circ)$	
	$= -\tan 17^\circ$	
	• $\cos(1530^\circ + x) = \cos(90^\circ + x)$ $\quad -4.360^\circ$	
	$= -\sin x$	
	$\therefore \dfrac{\sin^2 x + \frac{\tan 17^\circ}{-\tan 17^\circ}}{1 + (-\sin x)}$ ✓ ✓ ✓ ✓	
	$= \dfrac{\sin^2 x - 1}{1 - \sin x}$	
	$= \dfrac{(\sin x + 1)(\sin x - 1)}{-(\sin x - 1)}$ ✓	
	$= -(\sin x + 1)$	
	$= -\sin x - 1$ ✓	6
7.5.	$7\sin^2 x + 4\sin x \cos x - 4$	
	$= 7\sin^2 x + 4\sin x \cos x - 4 \cdot 1$	
	$= 7\sin^2 x + 4\sin x \cos x - 4(\sin^2 x + \cos^2 x)$ ✓	
	$= 7\sin^2 + 4\sin x \cos x - 4\sin^2 x - 4\cos^2 x$	
	$= 3\sin^2 x + 4\sin x \cos x - 4\cos^2 x$ ✓	
	$= (3\sin x - 2\cos x)(\sin x + 2\cos x)$ ✓	3

7.6.1. LHS $= \cos x \left(\tan x + \frac{1}{\tan x}\right)$

$= \cos x \left(\frac{\sin x}{\cos x} + \frac{1}{\frac{\sin x}{\cos x}}\right)$

$= \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$ ✓

$= \cos x \left(\frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}\right)$ ✓ nvm ✓ dem

$= \frac{1}{\sin x}$ $s^2 + c^2 = 1$ ✓

$=$ RHS 4

7.6.2 • $\tan x = UD$ ✓

$\frac{\sin x}{\cos x} = UD$

$\cos x = 0$

$x = 90° + k.180°; k \in \mathbb{Z}$ ✓

• $\tan x = 0$ ✓

$\frac{\sin x}{\cos x} = 0$

$\sin x = 0$

$x = k.180°; k \in \mathbb{Z}$ ✓ 4

• $\sin x = 0$

done!

QUESTION 8

8.1.	$x = 25° + k\,70°\ ;\ k \in \mathbb{Z}$	
	✓A	
	$x = -115°;\ -45°;\ 25°;\ 95°$ or $165°$	1
8.2.1.	$2\cos x = -\sin 1040°$	
	$\cos x = 0{,}32\ldots$ ✓	
	$\text{ref}\,\hat{} = 71{,}25\ldots°$	
	$\cos$ + in	
	I: $x = 71{,}25° + k\cdot 360°;\ k \in \mathbb{Z}$ ✓	
	or	
	IV: $x = 288{,}75° + k\,360°;\ k \in \mathbb{Z}$ ✓	3
8.2.2.	$3\sin 5x + \cos 5x = 0$	
	$3\sin A + \cos A = 0$ $\quad A = 5x$	
	$3\sin A = -\cos A$	
	$\frac{3\sin A}{\cos A} = \frac{-\cos A}{\cos A}$	
	$3\tan A = -1$	
	$\tan A = -\frac{1}{3}$ ✓	
	$\text{ref}\,\hat{} = 18{,}43\ldots°$	
	tan − in	
	II: $A = 161{,}56\ldots° + k\,180°$	
	$5x =$	

$\therefore \quad x = 32,31^\circ + k.36^\circ ; k \in \mathbb{Z}$ — 3

8 2.3. $\sin(x+10^\circ) + \cos 2(x-15^\circ) = 0$

$A = x+10^\circ \qquad B = 2(x-15^\circ)$

$\sin A + \cos B = 0$

$\sin A = -\cos B$

$\sin(270^\circ - B)$ — III $\qquad$ $\sin(270^\circ + B)$ — IV

$\sin A = \sin(270^\circ - B)$ or $\sin A = \sin(270^\circ + B)$

$A = 270^\circ - B + k360^\circ$	$A = 270^\circ + B + k.360^\circ$
$x+10^\circ = 270^\circ - (2x-30^\circ) + k.360^\circ$	$x+10 = 270^\circ + 2x - 30^\circ + k.360^\circ$
$x+10^\circ = 270^\circ - 2x + 30^\circ + k.360^\circ$	$-x = 230^\circ + k.360^\circ$
$3x = 290^\circ + k.360^\circ$	$x = -230^\circ - k.360^\circ ;$
$x = 96,67^\circ + k.120^\circ ;$	$k \in \mathbb{Z}$
$k \in \mathbb{Z}$	

4

8.2.4. $-3\tan x = 2\cos x$

$-3\frac{\sin x}{\cos x} = 2\cos x$

LCD $= \cos x \quad (\because \cos x \neq 0)$

x thru

$-3\sin x = 2\cos^2 x$ ✓

$= 2(1-\sin^2 x)$ ✓

$= 2 - 2\sin^2 x$

$2\sin^2 x - 3\sin x - 2 = 0$ ✓

$(\sin x - 2)(2\sin x + 1) = 0$ ✓

$\sin x = 2$ or $\sin x = -\frac{1}{2}$ ✓ both

no soln ref$^\wedge = 30°$

✓ sin − in

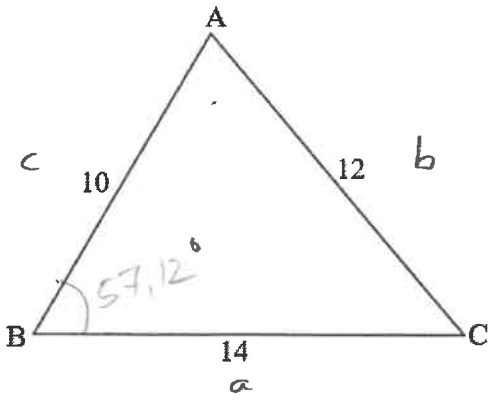
III : $x = 210° + k\,360°;\ k \in \mathbb{Z}$

or

IV : $x = 330° + k\,360°;\ k \in \mathbb{Z}$ 8

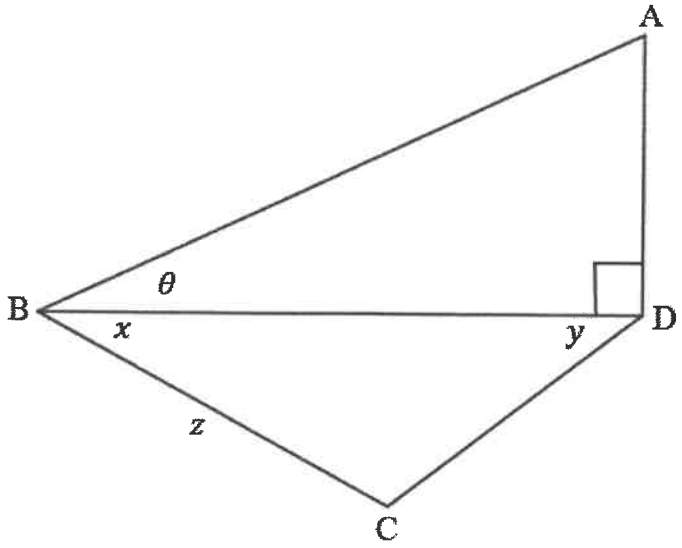
QUESTION 9

9.1.



9.1.1.	$12^2 = 10^2 + 14^2 - 2.10.14.\cos\hat{B}$ ✓	
	$-152 = -280.\cos\hat{B}$	
	$\frac{19}{35} = \cos\hat{B}$ ✓	
	ref $\hat{}$ = 57,12... °	
	Cos + in	
	I : $\hat{B} = 57,12°$ ✓	3
9.1.2.	area ΔABC = ✓ $\frac{1}{2}(10)(14)\sin 57,12°$	
	= ✓ 58,79	2

9.2.



$\hat{C} = 180° - (x+y)$ sum ^'s in $\Delta = 180°$

In ΔBCD : $\frac{BD}{\sin(180° - (x+y))} = \frac{z}{\sin y}$

$\frac{BD}{\sin(x+y)} = \frac{z}{\sin y}$

$\therefore BD = \frac{z \sin(x+y)}{\sin y}$

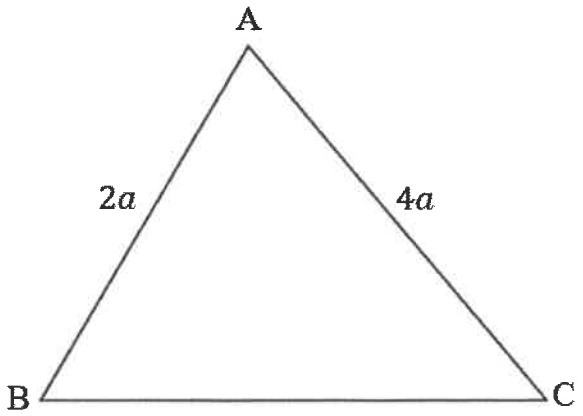
In ΔABD : $\frac{AD}{BD} = \tan\theta$

$AD = BD \cdot \tan\theta$

$= \frac{z \sin(x+y)}{\sin y} \cdot \tan\theta$

$= \frac{z \tan\theta \sin(x+y)}{\sin y}$ 4

9.3.



9.3.1.	$\hat{A} = 180° - x + y$	
	$= 180° - (x - y)$ ✓	
	$BC^2 = (2a)^2 + (4a)^2 - 2(2a)(4a)\cos(180° - (x-y))$ ✓	
	$= 4a^2 + 16a^2 - 16a^2(-\cos(x-y))$ ✓	
	$= 20a^2 + 16a^2\cos(x-y)$	
	$= 4a^2(5 + 4\cos(x-y))$	
	$BC = \sqrt{4a^2(5 + 4\cos(x-y))}$ ✓ cf +	
	$= \sqrt{4a^2} \cdot \sqrt{5 + 4\cos(x-y)}$	
	$= 2a\sqrt{5 + 4\cos(x-y)}$	4
9.3.2.	$BC = 2(10)\sqrt{5 + 4\cos(50° - 12°)}$ ✓	
	$= 57,10$ ✓	2